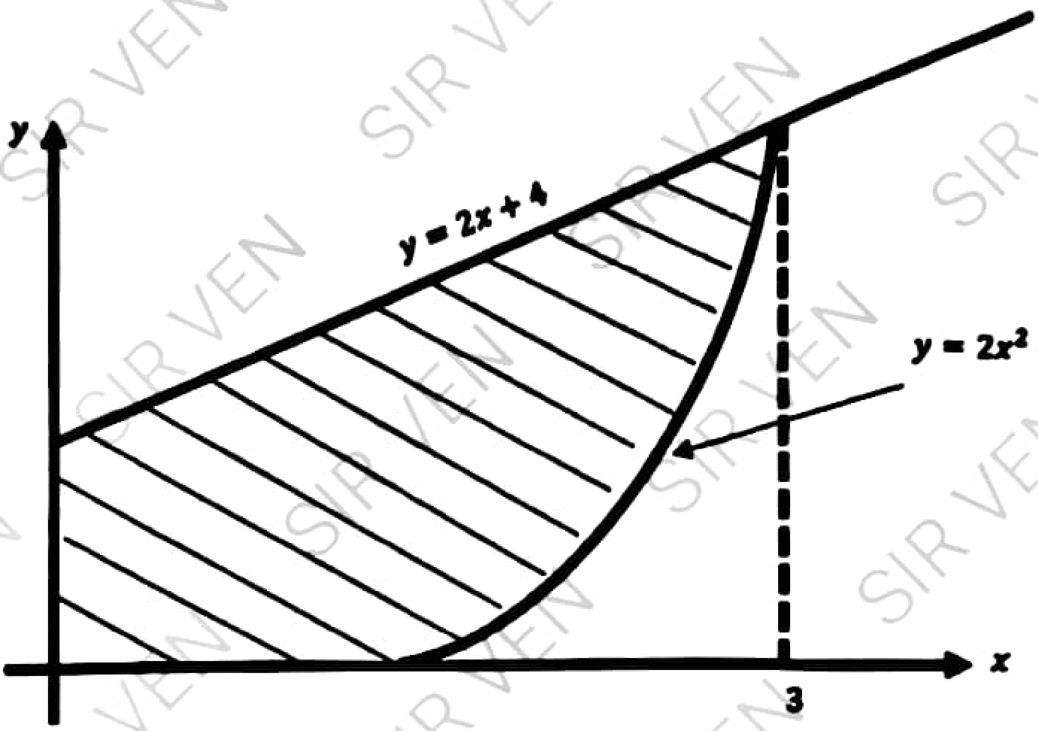
TRIAL NEGERI 2022 INTEGRATION OR PENGAMIRAN KSSM SPM ADD MATH

SIR VEN

Q1: PERLIS

Rajah 9 menunjukkan sebahagian daripada lengkung $y = 2x^2 - 8$ dan garis lurus $y = 2x + 4$.

Diagram 9 shows part of a curve $y = 2x^2 - 8$ and a straight line $y = 2x + 4$.



Rajah 9/ ***Diagram 9***

(a) Hitung luas rantau berlorek.

Calculate the area of the shaded region.

[6 markah/6 *marks*]

(b) Rantau yang dibatasi oleh lengkung, paksi-x, paksi-y dan garis lurus $y = h$ diputarkan melalui 360° pada paksi-y. Cari isi padu kisaran, dalam sebutan π.

The region enclosed by the curve, the x-axis, the y-axis and the straight line $y = h$ is revolved through 360° about the y-axis. Find the of revolution, in terms of π.

[4 markah/4 *marks*]

9

(a) $x = 2$ atau $y = 10$ $\boxed{P1}$ dilihat

Cari luas trapezium K1

$\frac{1}{2}(4+10)3$

Kamirkan $\int 2x^2 - 8\, dx$ K1

$\left[\frac{2x^3}{3} - 8x\right]$

Ganti had $\int_2^3$ ke dalam $\int 2x^2 - 8\, dx$ K1

$\left(\frac{2(3)^3}{3} - 8(3)\right) - \left(\frac{2(2)^3}{3} - 8(2)\right)$

Luas trapezium – luas dibawah graf K1

$21 - \frac{14}{3}$

$\boxed{N1}$ $\frac{49}{3}$

(b) $h = 4$ $\boxed{P1}$ dilihat

Kamirkan $\int \pi x^2\, dy$ K1

$\left[\frac{y^2}{4} + 4y\right]$

Guna had $\int_0^4$ *kedalam* $\left[\frac{y^2}{4} + 4y\right]$ K1

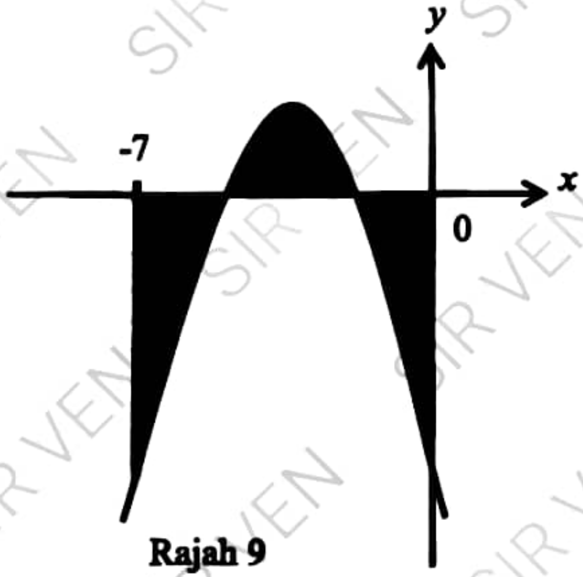
$\pi\left[\frac{(4)^2}{4} + 4(4) - 0\right]$

$\boxed{N1}$ 20π

Q2: SABK (ASRAMA)

Rajah 9 menunjukkan suatu lengkung $y = f(x)$ di mana $f(x) = -x^2 - 7x - 10$.

Diagram 9 shows a curve $y = f(x)$ where $f(x) = -x^2 - 7x - 10$.



Rajah 9

Diagram 9

Hitung

Calculate

(a) jumlah luas kawasan berlorek, **[6 *markah*]**

the total area of the shaded regions, **[6 *marks*]**

(b) isipadu kisaran, dalam sebutan π, apabila rantau berlorek A dikisarkan melalui 360° pada paksi-x.

the volume of revolution, in terms of π, when the shaded region A is revolved through 360° about the x-axis. **[4 *markah*]**

[4 *marks*]

(a) Cari had / punca DAN pemfaktoran

Find the limits / roots AND factorisation

$$f(x)=(-x-2)(x+5)$$

Pengamiran mana-mana dua luas

Integration any two areas

$$\left|\int_{-7}^{-5} -x^2-7x-10\,dx\right|=\left|\left[-\frac{x^3}{3}-\frac{7x^2}{2}-10x\right]_{-7}^{-5}\right|$$

@

$$\left|\int_{-5}^{-2} -x^2-7x-10\,dx\right|=\left|\left[-\frac{x^3}{3}-\frac{7x^2}{2}-10x\right]_{-5}^{-2}\right|$$

@

$$\left|\int_{-2}^{0} -x^2-7x-10\,dx\right|=\left|\left[-\frac{x^3}{3}-\frac{7x^2}{2}-10x\right]_{-2}^{0}\right|$$

Ganti had ke dalam mana-mana fungsi

Substitute limit to any functions

$$\left(-\frac{(-5)^3}{3}-\frac{7(-5)^2}{2}-10(-5)\right)-\left(-\frac{(-7)^3}{3}-\frac{7(-7)^2}{2}-10(-7)\right)=\frac{26}{3}$$

@

$$\left(-\frac{(-2)^3}{3}-\frac{7(-2)^2}{2}-10(-2)\right)-\left(-\frac{(-5)^3}{3}-\frac{7(-5)^2}{2}-10(-5)\right)=\frac{9}{2}$$

@

$$\left(-\frac{(0)^3}{3}-\frac{7(0)^2}{2}-10(0)\right)-\left(-\frac{(-2)^3}{3}-\frac{7(-2)^2}{2}-10(-2)\right)=\frac{26}{3}$$

Jumlah luas / *Total area*

$$=\frac{26}{3}+\frac{9}{2}+\frac{26}{3}$$

$$=\frac{131}{6}$$

(b)

$$=\pi\left|\int_{-5}^{-2}\left(-x^2-7x-10\right)^2 dx\right|$$

$$=\pi\left|\int_{-5}^{-2} x^4+14x^3+59x^2+150x+100\,dx\right|$$

Pengamiran / *Integration*

$$=\pi\left(\frac{x^5}{5}+\frac{7x^4}{2}+\frac{59x^3}{3}+75x^2+100x\right)_{-5}^{-2}$$

Ganti had / *Substitute limits*

$$=\pi\left[\left(\frac{(-2)^5}{5}+\frac{7(-2)^4}{2}+\frac{59(-2)^3}{3}+75(-2)^2+100(-2)\right)-\left(\frac{(-5)^5}{5}+\frac{7(-5)^4}{2}+\frac{59(-5)^3}{3}+75(-5)^2+100(-5)\right)\right]$$

$$=\frac{81}{10}\pi$$

Q3: MELAKA

(a) Suatu lengkung dengan fungsi kecerunan $2x - \frac{2}{x^2}$ mempunyai titik pegun di (k, 8).

A curve with gradient function $2x - \frac{2}{x^2}$ *has a stationary point at* (k, 8).

(i) Cari nilai k [2 markah]

Find the value of k [2 *marks*]

(ii) Seterusnya , dengan kaedah melakar garis tangent ,tentukan sama ada titik pegun ini titik maksimum atau titik minimum dengan memberi justifikasi anda,

[3 markah]

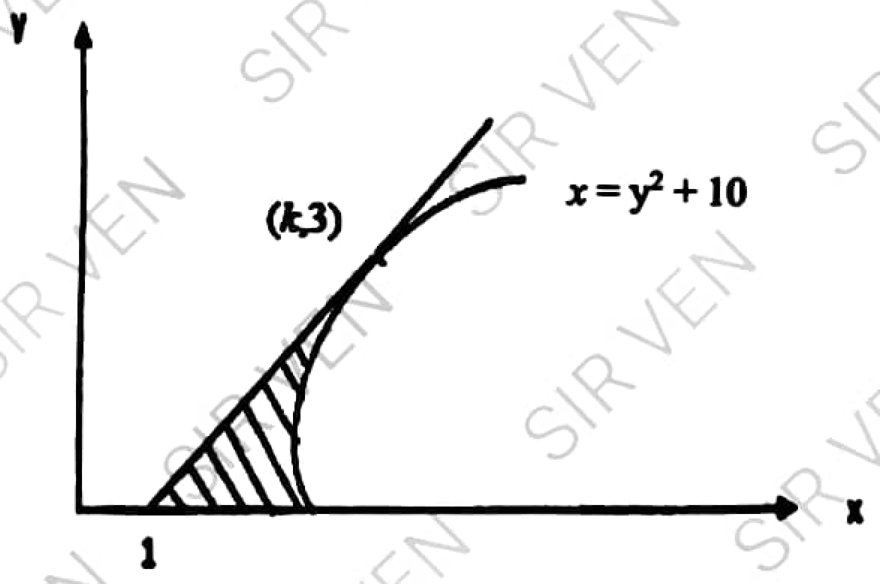
Hence, by using sketching of tangent method , determine whether the stationary point is a maximum or a minimum point by giving your justification [3 *marks*]

(b) Rajah 4 di bawah menunjukkan garis lurus yang mempunyai x-pintasan sama dengan 1 menyentuh garis lengkung $x = y^2 + 10$ pada titik (k, 3) . Diberi luas kawasan berlorek ialah 9 $unit^2$. Dengan menggunakan kaedah pengamiran , cari nilai bagi k.

[5 markah]

Diagram 4 below shows a straight line with x-intercept 1 touches a curve $x = y^2 + 10$ *at point* (k,3) . *Given the area of shaded region is* 9 $unit^2$. *By using integration method, find the value of k.*

[5 *marks*]



Rajah 4 / *Diagram 4*

$$\frac{dy}{dx} = 2x - \frac{2}{x^2}$$

Pada titik pegun , $\frac{dy}{dx} = 0$, $x = k$, *maka*

$$2k - \frac{2}{k^2} = 0$$

$$k = 1$$

x	0.5	1	1.5
$\frac{dy}{dx}$	-7	0	$2\frac{1}{9}$
tangen tangent	$\setminus$	—	/
(1, 8) merupakan titik minimum.			

$\int_0^3 (y^2 + 10)dy$ *or* $\frac{1}{2}(1 + k)(3)$

$$\int_0^3 (y^2 + 10)dy - \frac{1}{2}(1 + k)(3) = 9$$

$$\left[\frac{y^3}{3} + 10y\right]_0^3 - \frac{3}{2} - \frac{3}{2}k = 9$$

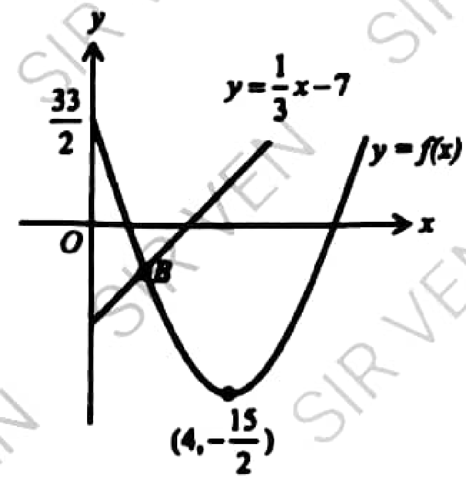
$$\left[\left(\frac{3^3}{3} + 10\text{x}\,3\right) - 0\right] - \frac{3}{2} - \frac{3}{2}k = 9$$

$k = 19$

Q4: TERENGGANU

Rajah 3 menunjukkan graf bagi suatu lengkung $y = f(x)$. Garis lurus $y = \frac{1}{3}x - 7$ adalah normal kepada lengkung itu pada titik B.

Diagram 3 shows a graph of a curve $y = f(x)$. A straight line $y = \frac{1}{3}x - 7$ is a normal to the curve at point B.



Rajah 3 / *Diagram 3*

Fungsi kecerunan bagi lengkung itu ialah $kx - 12$, dengan keadaan k ialah pemalar. Diberi bahawa lengkung itu mempunyai titik minimum pada $(4, -\frac{15}{2})$.

The gradient function of the curve is $kx - 12$, where k is a constant. Given that the curve has a minimum point at $(4, -\frac{15}{2})$.

(a) **Cari**
Find

(i) **nilai k.**
the value of k.

(ii) **koordinat titik B.**
the coordinates of point B.

[4 *markah*]
[4 *marks*]

(b) Cari persamaan bagi lengkung itu.
Find the equation of the curve.
arkah]
[2 *marks*]

(c) Jika lengkung itu digerakkan 4 unit ke kiri, cari persamaan baharu lengkung itu. Seterusnya, cari isipadu janaan, dalam sebutan π, apabila rantau yang dibatasi oleh lengkung itu dan garis lurus $y = -6$ dikisarkan 180° pada paksi-y. [4 *markah*]
If the curve moved 4 unit to the left, find the new equation of the curve. Hence, find the volume generated, in term of π, when the region bounded by the curve and the straight line $y = -6$ is revolved 180° about the y-axis. [4 *marks*]

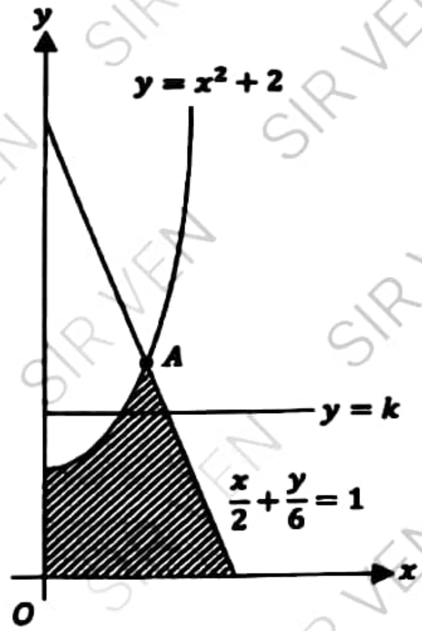
(a)	(i)	$k(4) - 12 = 0$	K1
		$k = 3$	N1
	(ii)	$m_1\left(\frac{1}{3}\right) = -1$ atau $3x - 12 = -3$	K1
		$(3, -6)$	N1

(b)	$y = \frac{3}{2}x^2 - 12x + c$	
	$-6 = \frac{3}{2}(3)^2 - 12(3) + c$	K1
	$y = \frac{3}{2}x^2 - 12x + \frac{33}{2}$	N1
(c)	$y = \frac{3}{2}x^2 - \frac{15}{2}$	P1
	$V = \pi \int_{-7.5}^{-6} \left(\frac{2}{3}y + 5\right) dy$	
	$\pi\left[\frac{2y^2}{2(3)} + 5y\right]_{-7.5}^{-6}$	K1
	$\pi\left[\left(\frac{2(-6)^2}{2(3)} + 5(-6)\right) - \left(\frac{2(-7.5)^2}{2(3)} + 5(-7.5)\right)\right]$	K1
	0.75π	N1

Q5: KELANTAN MIMS (SET 1)

Rajah 2 menunjukkan lengkung $y = x^2 + 2$ yang bersilang dengan garis lurus $\frac{x}{2} + \frac{y}{6} = 1$ pada titik A.

Diagram 2 shows the curve $y = x^2 + 2$ which is intersects the straight line $\frac{x}{2} + \frac{y}{6} = 1$ at point A.



Rajah 2

Diagram 2

(*a*) Cari koordinat titik A. **[2 *markah*]**

Find the coordinate of point A. **[2 *marks*]**

(*b*) Hitung luas rantau berlorek. **[3 *markah*]**

Calculate the area of shaded region. **[3 *marks*]**

(*c*) Diberi bahawa isipadu janaan apabila rantau yang dibatasi oleh lengkung $y = x^2 + 2$, paksi-*y* dan garis lurus $y = k$, dikisarkan melalui $360°$ pada paksi-*y* ialah $\frac{\pi}{8}$. Cari nilai k. **[3 *markah*]**

It is given that the volume generated when the region which is bounded by the curve $y = x^2 + 2$, the y-axis and the straight line $y = k$, is revolved through $360°$ about the y-axis is $\frac{\pi}{8}$. Find the value of k. **[3 *marks*]**

Jawapan / *Answer:*

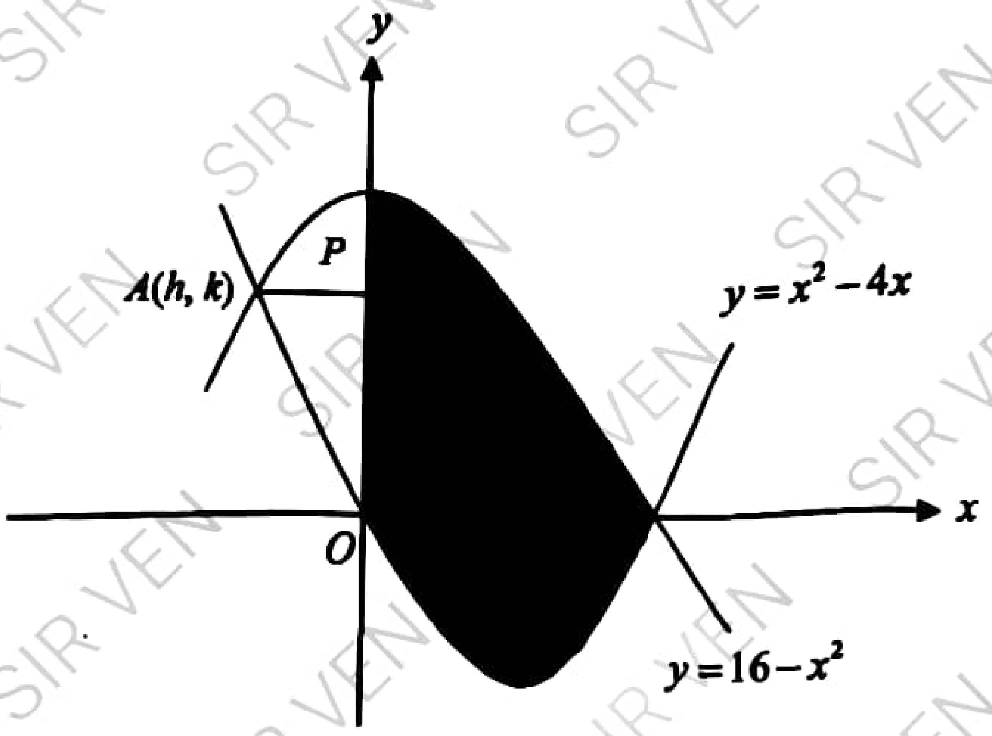
5(a)	$\frac{x}{2}+\frac{x^2+2}{6}=1$ $A(1,3)$
(b)	$\left[\frac{x^3}{3}+2x\right]_0^1$ or $\frac{1}{2}(1)(3)$ $\left[\left(\frac{(1)^3}{3}+2(1)\right)-\left(\frac{0}{3}+2(0)\right)\right]+\frac{3}{2}$ $\frac{23}{6}$ // 3.833
(c)	$\pi\left[\frac{y^2}{2}-2y\right]_2^k=\frac{\pi}{8}$ $\pi\left[\left(\frac{k^2}{2}-2k\right)-\left(\frac{2^2}{2}-2(2)\right)\right]=\frac{\pi}{8}$ $k=2.5$

CamScanner

Q6: KELANTAN MIMS (SET2)

Rajah 6 menunjukkan lengkung $y = 16 - x^2$ dan $y = x^2 - 4x$.

Diagram 6 shows the curve $y = 16 - x^2$ and $y = x^2 - 4x$.



Rajah 6

Diagram 6

Cari

Find

(*a*) nilai h dan nilai k, [2 *markah*]

the value of h and of k, [2 *marks*]

(*b*) luas kawasan berlorek, [4 *markah*]

the area of the shaded region, [4 *marks*]

(*c*) isipadu yang dijanakan, dalam sebutan π, apabila rantau P dikisarkan 360° pada paksi -y. [4 *markah*]

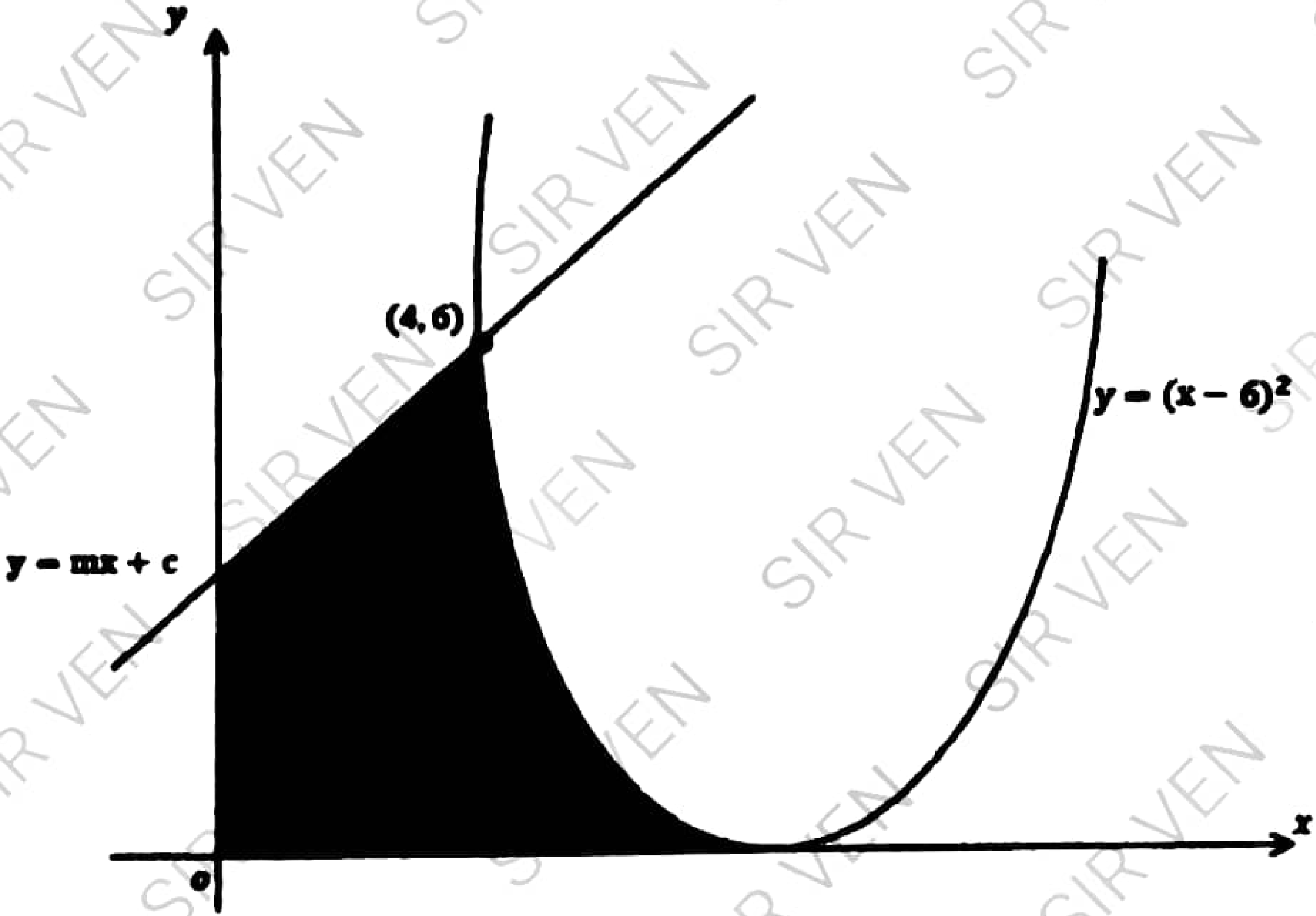
the volume generated, in terms of π, *when the region P is revolved through* 360° *about y- axis.* [4 *marks*]

8 (*a*)	$16-x^2=x^2-4x$ $x^2-2x-8=0$ $(x+2)(x-4)=0$ $x=-2, x=4$ Di titik A, $y=16-(-2)^2=12$ $h=-2, k=12$	2
(*b*)	$\int_0^4 (16x-x^2)dx+\int_0^4 (x^2-4x)dx$ $\left[16x-\frac{x^3}{3}\right]_0^4+\left[\frac{x^3}{3}-\frac{4x^2}{2}\right]_0^4$ $\left[\left(16(4)-\frac{(4)^3}{3}\right)-\left(16(0)-\frac{(0)^3}{3}\right)\right]+\left[\left(\frac{(4)^3}{3}-\frac{4(4)^2}{2}\right)-\left(\frac{(0)^3}{3}-\frac{4(0)^2}{2}\right)\right]$ $\frac{128}{3}+\left\|-\frac{32}{3}\right\|$ $\frac{160}{3}$ // $53\frac{1}{3}$ //53.33 unit^2	4
(*c*)	$Isipadu=\pi\int_{12}^{16}(16-y)dy$ $=\pi\left[\left(16y-\frac{y^2}{2}\right)\right]_{12}^{16}$ $=\pi\left[\left(16(16)-\frac{(16)^2}{2}\right)-\left(16(12)-\frac{(12)^2}{2}\right)\right]$ $=8\pi$ *unit*^{3}	4

Q7: PAHANG

Rajah 3 menunjukkan sebahagian daripada lengkung $y = (x-6)^2$ dan garis lurus $y = mx + c$ bersilang pada titik $(4, 6)$. Diberi $\int_0^4 (mx + c)\, dx = 16$.

Diagram 3 show part of a curve $y = (x-6)^2$ and a straight line $y = mx + c$ intersect at point $(4, 6)$. Given that $\int_0^4 (mx + c)\, dx = 16$.



Rajah 3 / *Diagram 3*

Hitung/Calculate

(a) **nilai c dan nilai m** **[4 markah]**
the value of c and of m **[4 *marks*]**

(b) **Luas kawasan berlorek** **[3 markah]**
The area of shaded region **[3 *marks*]**

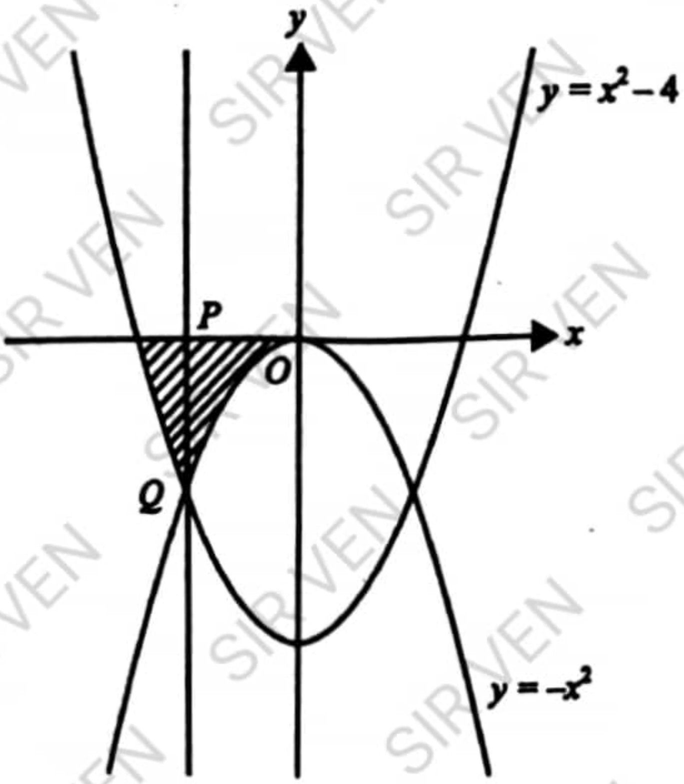
(c) **Isipadu yang dijanakan dalam sebutan π apabila rantau berlorek dikisarkan melalui pada paksi -x** **[3 markah]**
The volume generated in term of π when the shaded region is revolved through 360^0 about x-axis. **[3 *marks*]**

(a)	$\left[\frac{mx^2}{2} + Cx\right]_0^4 = 16$	K1
	$y = mx + c$ Gunakan koordinat/*use coordinate* $(4, 6)$	
	$6 = m(4) + C$	K1
	Selesaikan menggunakan kaedah persamaan serentak	
	$m = 1$ dan $c = 2$	N1 N1
(b)	Luas Berlorek = Luas trapezium + Luas Di bawah Lengkung *Shaded area = Area of trapezium + Area under curve*	
	$= \frac{1}{2}(6+2)(4)$ K1 OR $\left[\frac{(x-6)^3}{3(1)}\right]_4^6$ $K1$	
	$= 16 + \frac{8}{3}$	K1
	$= 18\frac{2}{3}$ @ $\frac{56}{3}$ @ 18.667	N1
(c)	Jumlah isipadu = Isipadu janaan oleh garis lurus + Isipadu janaan oleh lengkung *Total Volume = Volume generated straight line + Volume generated curve* Use limit	
	$\pi\left[\frac{(x+2)^3}{3(1)}\right]_0^4$	K1
	OR	
	$\pi\left[\frac{(x-6)^5}{5(1)}\right]_4^6$	K1
	$\frac{208}{3}\pi + \frac{32}{5}\pi$	
	$75\frac{11}{15}$ @ $\frac{1136}{15}$ @ 75.733	N1

Q8: SELANGOR(SET 2)

Rajah 4 menunjukkan dua lengkung $y = -x^2$ dan $y = x^2 - 4$ bersilang pada titik Q. Garis lurus PQ selari dengan paksi-y.

Diagram 4 shows two curves $y = -x^2$ and $y = x^2 - 4$ are intersecting at point Q. The straight line PQ is parallel to the y-axis.



Rajah 4
Diagram 4

Cari
Find

(a) **persamaan garis lurus PQ,**
the equation of the straight line PQ,

[2 markah]
[2 marks]

(b) **luas, dalam unit^2, bagi rantau berlorek,**
area, in unit^2, of the shaded region,

[3 markah]
[3 marks]

(c) **isi padu yang dijanakan apabila rantau yang dibatasi oleh lengkung $y = -x^2$ dan lengkung $y = x^2 - 4$ diputarkan melalui 180° pada paksi-y.**
the volume generated when the region bounded by the curves $y = -x^2$ and $y = x^2 - 4$ is rotated 180° about the y-axis.

[3 markah]
[3 marks]

(a) $-x^2 = x^2 - 4$

$x = -\sqrt{2}$

(b) $-\int_{-2}^{-\sqrt{2}} x^2 - 4\,dx - \int_{-\sqrt{2}}^{0} -x^2\,dx$

$= -\left[\frac{x^3}{3} - 4x\right]_{-2}^{-\sqrt{2}} - \left[-\frac{x^3}{3}\right]_{-\sqrt{2}}^{0}$

$= -\left[\left(\frac{(-\sqrt{2})^3}{3} - 4(-\sqrt{2})\right) - \left(\frac{(-2)^3}{3} - 4(-2)\right)\right] - \left[0 - \left(-\frac{(-\sqrt{2})^3}{3}\right)\right]$

$= \frac{16 - 8\sqrt{2}}{3} = 1\cdot 562$

(c) $= \pi\left[\frac{y^2}{2} + 4y\right]_{-4}^{-2} + \pi\left[-\frac{y^2}{2}\right]_{-2}^{0}$

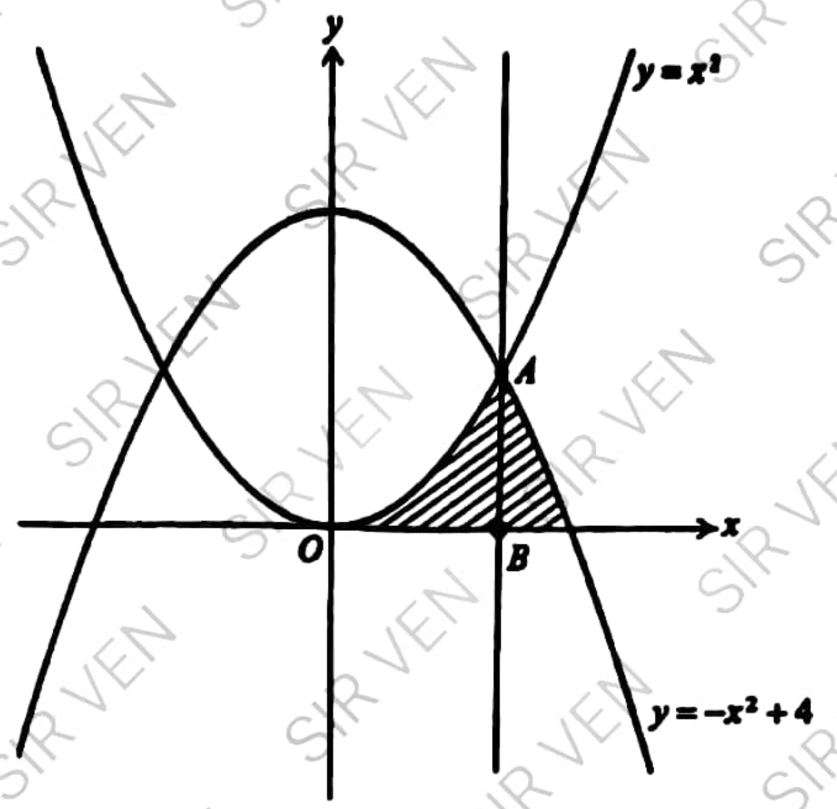
$= \pi\left[\left(\frac{(-2)^2}{2} + 4(-2)\right) - \left(\frac{(-4)^2}{2} + 4(-4)\right)\right] + \pi\left[-0 - \left(-\frac{(=2)^2}{2}\right)\right]$

$= 4\pi$ unit^3

Q9: SELANGOR (SET 1)

Rajah 4 menunjukkan dua lengkung $y = x^2$ dan $y = -x^2 + 4$ bersilang pada titik A. Garis lurus AB selari dengan paksi-y.

Diagram 4 shows two curves $y = x^2$ and $y = -x^2 + 4$ are intersecting at point A. The straight line AB is parallel to the y-axis.



Rajah 4

Diagram 4

Cari

Find

(a) **persamaan garis lurus AB,**
the equation of the straight line AB,

[2 markah]
[2 marks]

(b) **luas, dalam unit^2, bagi rantau berlorek,**
area, in unit^2, of the shaded region,

[3 markah]
[3 marks]

(c) **isi padu yang dijanakan apabila rantau yang dibatasi oleh lengkung $y = x^2$ dan lengkung $y = -x^2 + 4$ dikisarkan 180° pada paksi-y.**
the volume generated when the region bounded by the curves $y = x^2$ and $y = -x^2 + 4$ is revolved 180° about the y-axis.

[3 markah]
[3 marks]

(a) $x^2 = -x^2 + 4$

$x = \sqrt{2}$

(b) $\int_0^{\sqrt{2}} x^2 \ dx + \int_{\sqrt{2}}^{2} -x^2 + 4 \ dx$

$= \left[\frac{x^3}{3}\right]_0^{\sqrt{2}} + \left[\frac{-x^3}{3} + 4x\right]_{\sqrt{2}}^{2}$

$= \left[\frac{\sqrt{2}^3}{3} - 0\right] + \left[\left(-\frac{2^3}{3} + 4(2)\right) - \left(-\frac{\sqrt{2}^3}{3} + 4\sqrt{2}\right)\right]$

$= \frac{16-8\sqrt{2}}{3}$ unit^2 or $1{\cdot}562$ unit^2

(c) $\pi \int_2^4 4 - y \, dy \ + \pi \int_0^2 y \, dy$

$= \pi \left[4y - \frac{y^2}{2}\right]_2^4 + \pi \left[\frac{y^2}{2}\right]_0^2$

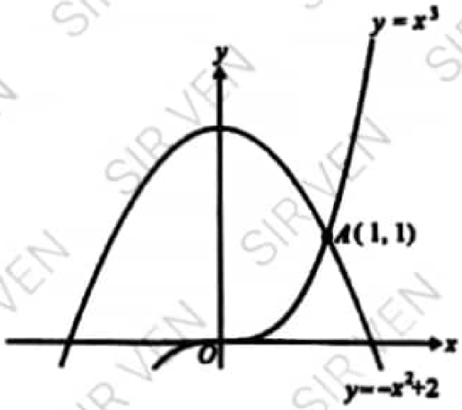
$= \pi \left[\left(4(4) - \frac{4^2}{2}\right) - \left(4(2) - \frac{2^2}{2}\right)\right] + \pi \left[\frac{2^2}{2} - 0\right]$

$= 4\pi$ unit^3

Q10: KEDAH

a) Rajah 8(a) menunjukkan garis lengkung $y = x^3$ dan $y = -x^2 + 2$ yang bersilang pada titik A.

Diagram 8(a) shows the lines of the curve $y = x^3$ and $y = -x^2 + 2$ which intersect at point A.



Rajah 8(a)
Diagram 8(a)

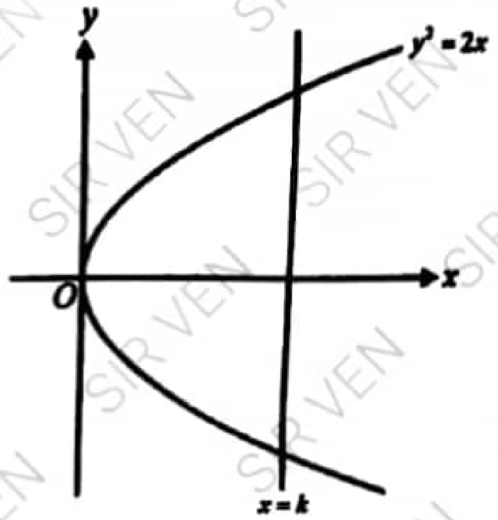
(i) Nyatakan lengkung yang manakah menghasilkan kecerunan < 0 di titik A. Tunjukkan pengiraan anda.
State which curve produces a gradient < 0 at point A. Show your calculation.

[3 markah / *marks*]

(ii) Cari luas yang dibatasi oleh lengkung $y = x^3$, $y = -x^2 + 2$ dan paksi-y.
Find the area bounded by the curve of $y = x^3$, $y = -x^2 + 2$ and y-axis.

[4 markah / *marks*]

(b) Rajah 8(b) menunjukkan lengkung $y^2 = 2x$ bersilang pada garis lurus $x = k$.
Diagram 8 (b) shows an intersecting curve $y^2 = 2x$ at a straight line $x = k$.



Rajah 8(b)
Diagram 8(b)

Cari nilai k apabila isipadu janaan bagi rantau yang dibatasi oleh lengkung $y^2 = 2x$, $x = k$ dan paksi-x dikisarkan 180^0 pada paksi -x ialah 9π unit^3.
Find the value of k when the volume generated bounded by the curve $y^2 = 2x$, $x = k$ and the x-axis rotated 180^0 on the -x axis is 9π unit^3.

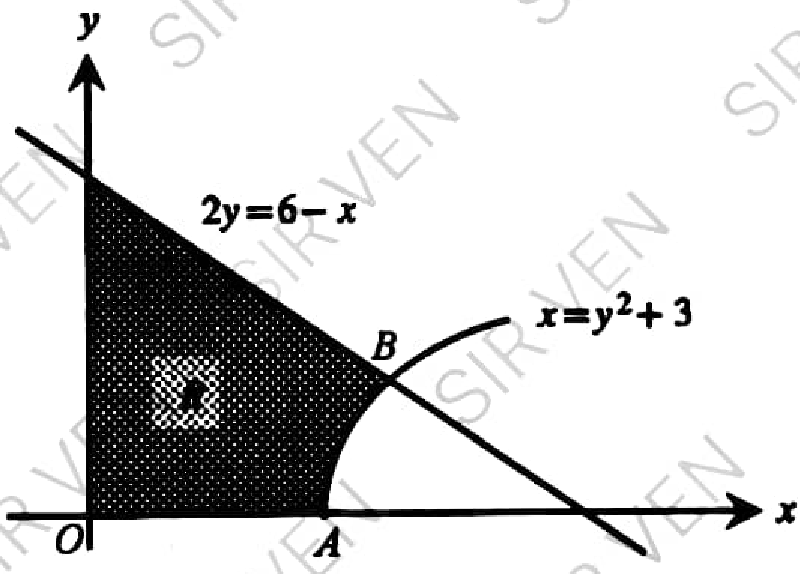
[3 markah / *marks*]

(a)(i)	$\frac{dy}{dx}=3x^2$ atau $\frac{dy}{dx}=-2x$ K1 -2 N1 Lengkung $y=-x^2+2$ N1
(a)(ii)	$\int_0^1 -x^2+2\ dx-\int_0^1 x^3\ dx$ K1 $\left[-\frac{x^3}{3}+2x\right]_0^1-\left[\frac{x^4}{4}\right]_0^1$ K1 $\left(\left[-\frac{(1)^3}{3}+2(1)\right]-\left[-\frac{(0)^3}{3}+2(0)\right]\right)-\left(\left[\frac{(1)^4}{4}\right]-\left[\frac{0^4}{4}\right]\right)$ K1 $\frac{17}{12}$ unit^2 N1
(b)	$\pi\int_0^k 2x\ dx=9\pi$ $\left[\frac{2x^2}{2}\right]_0^k=9$ K1 $(k^2)-(0)=9$ K1 $k=3$ N1

Q11: PERAK

Rajah 5 menunjukkan lengkung $2y = 6 - x$ bersilang dengan garis lurus $x = y^2 + 3$ pada titik B dan paksi-x pada titik A.

Diagram 5 shows the curve $2y = 6 - x$ intersects the straight line $x = y^2 + 3$ at B and the x-axis at point A.



Rajah 5

Diagram 5

Cari

Find

(*a*) koordinat titik *B*, [3 *markah*]

the coordinates of point B, [3 *marks*]

(*b*) luas rantau berlorek R, [4 *markah*]

the area of the shaded region R, [4 *marks*]

(*c*) isipadu yang dijanakan, dalam sebutan π, apabila rantau yang dibatasi oleh lengkung $x = y^2 + 3$, garis lurus $x = 4$ dan paksi -x dikisarkan melalui $360°$ pada paksi-x.

the volume generated, in terms of π, when the region bounded by the curve $x = y^2 + 3$, the straight line $x = 4$ and the x-axis is revolved through $360°$ about the x-axis.

[3 *markah*]

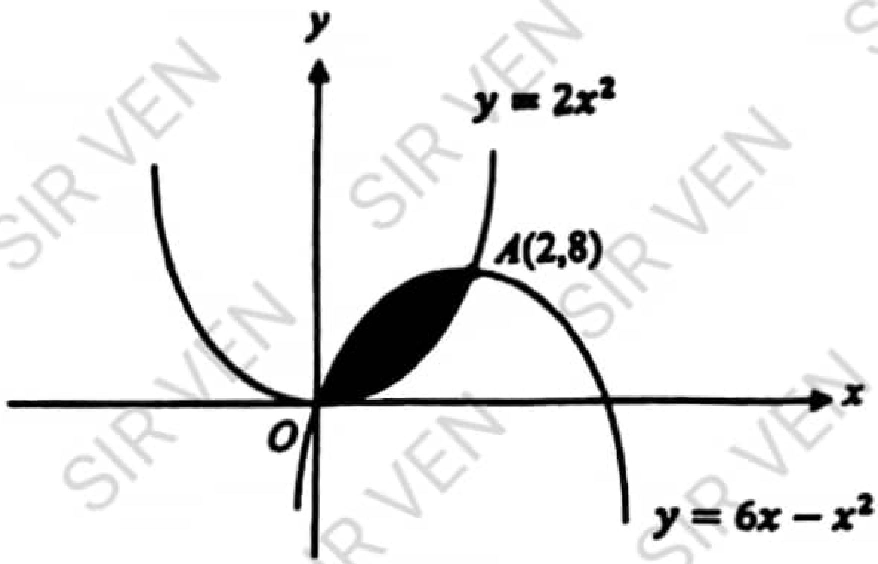
[3 *marks*]

8(*a*)	$2y = 6-(y^2+3)$ $y=1,\ y=-2$ (*abaikan*) $(4,1)$
(*b*)	$\frac{y^3}{3}+3y$ $\frac{1}{2}\times 2\times 4$ atau Guna $\int_0^1$ dalam $\frac{y^3}{3}+3y$ $\frac{1}{2}\times 2\times 4 + \int_0^1 y^2+3\ dy$ $\frac{22}{3}$ // $7\frac{1}{3}$
(*c*)	$\pi\int_3^4 (x-3)\ dx$ $\pi\left[\frac{(4)^2}{2}-3(4)\right]-\left[\frac{(3)^2}{2}-3(3)\right]$ $\frac{1}{2}\pi$

Q12: NEGERI SEMBILAN

Rajah 4 menunjukkan suatu kawasan berlorek yang dibatasi di antara dua lengkung $y = 2x^2$ dan $y = 6x - x^2$. Kedua-dua lengkung itu bersilang pada asalan dan titik $A(2,8)$.

Diagram 4 shows that a shaded region is bounded between two curves $y = 2x^2$ and $y = 6x - x^2$. Both curves intersect at the origin and point $A(2,8)$.



Rajah 4

Diagram 4

Hitung

Calculate

(a) luas kawasan berlorek, [3 markah]

area of the shaded region, [3 marks]

(b) isi padu kisaran, dalam sebutan π, apabila kawasan yang dibatasi oleh lengkung $y = 2x^2$, garis lurus $x = 2$ dan paksi-x dikisarkan melalui 360^0 pada paksi-x. [3 markah]

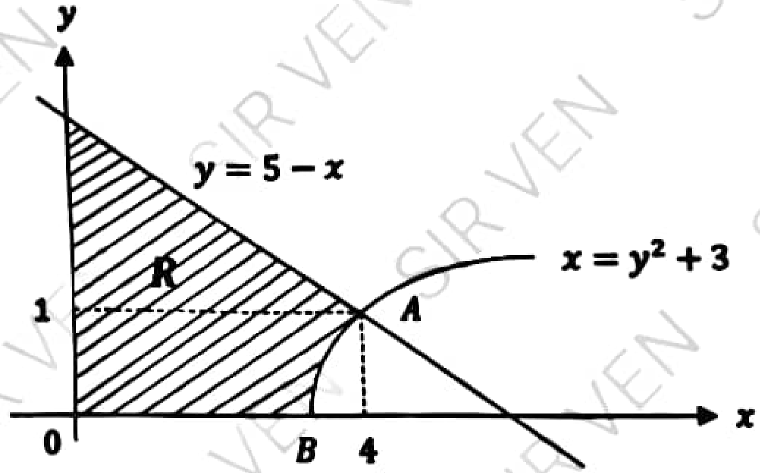
the volume of revolution, in terms of π , when the region bounded by the curve, the straight line $x = 2$ and the x-axis is revolved through 360^0 about the x-axis. [3 marks]

(a)	$\int_0^2 6x - x^2 - 2x^2 dx$ atau $\left[\frac{6x^2}{2} - \frac{3x^3}{3}\right]_0^2$ $\left[\frac{6(2)^2}{2} - \frac{3(2)^3}{3}\right] - 0$ 4
(b)	$\int_0^2 \pi(2x^2)^2 dx$ atau $\pi\left[\frac{4x^5}{5}\right]_0^2$ $\pi\left(\frac{4(2)^5}{5} - \frac{4(0)^5}{5}\right)$ $\frac{128}{5}\pi$

Q13: JOHOR

Rajah 8 menunjukkan lengkung $x = y^2 + 3$ bersilang dengan garis lurus $y = 5 - x$ pada $A(1,4)$ dan paksi-x pada titik B.

Diagram 8 shows the curve $x = y^2 + 3$ intersects the straight line $y = 5 - x$ at $A(1,4)$ and the x-axis at point B.



Rajah 8

Diagram 8

Cari

Find

(a) **koordinat titik B.**
the coordinates of point B.

(b) **luas rantau berlorek R.**
the area of the shaded region R.

(c) **isi padu yang dijanakan, dalam sebutan π, apabila rantau yang dibatasi oleh lengkung $x = y^2 + 3$, garis lurus $y = 5 - x$ dan paksi-x dikisarkan melalui 360° pada paksi-x.**
the volume generated, in terms of π, when the region bounded by the curve $x = y^2 + 3$, the straight line $y = 5 - x$ and the x-axis is revolved through 360° about the x-axis.

[10 markah/ *marks*]

Jawapan / *Answer*:

(a) Pada titik B, $y = 0$

$$x = (0)^2 + 3$$
$$x = 3$$

Maka, titik B ialah (3,0).

(b) Luas rantau berlorek R$= \frac{1}{2} \times 4 \times 4 + \int_0^1 x\, dy$

$$= \frac{1}{2} \times 4 \times 4 + \int_0^1 (y^2 + 3)\, dy$$

$$= 8 + \left[\frac{y^3}{3} + 3y\right]_0^1$$

$$= 8 + \left[\frac{(1)^3}{3} + 3(1) - \left(\frac{(0)^3}{3} + 3(0)\right)\right]$$

$$= 11\frac{1}{3}$$

(c) Isi padu janaan$= \frac{1}{3} \times \pi \times (1)^2 \times 1 + \int_3^4 \pi y^2 \; dx$

$$= \frac{1}{3} \times \pi \times (1)^2 \times 1 + \int_3^4 \pi(x - 3) \; dx$$

$$= \frac{1}{3}\pi + \pi\left[\frac{x^2}{2} - 3x\right]_3^4$$

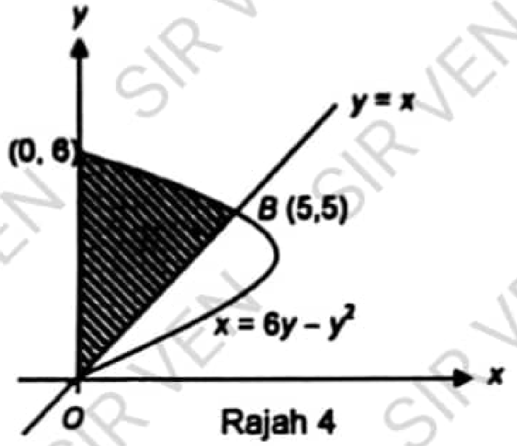
$$= \frac{1}{3}\pi + \pi\left[\frac{(4)^2}{2} - 3(4) - \left(\frac{(3)^2}{2} - 3(3)\right)\right]$$

$$= \frac{1}{3}\pi + \frac{1}{2}\pi$$
$$= \frac{5}{6}\pi$$

Q14: KELANTAN

Rajah 4 menunjukkan sebahagian daripada lengkung $x = 6y - y^2$ dan sebahagian daripada garis $y = x$ yang bersilang pada titik *B* dan titik *O*.

Diagram 4 shows part of curve $x = 6y - y^2$ and part of line $y = x$ that intercept at point B and point O.

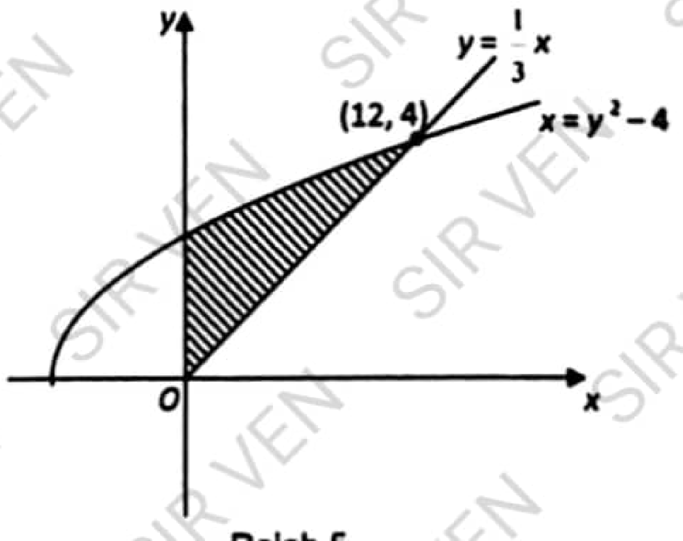


Rajah 4

Diagram 4

(*a*) Cari luas kawasan yang berlorek. [5 *markah*]

Find the area of shaded region. [5 *marks*]



Rajah 5

Diagram 5

(*b*) Berdasarkan Rajah 5, hitungkan isipadu yang dijanakan apabila rantau berlorek itu dikisarkan 360° pada $paksi - x$. [5 *markah*]

Based on Diagram 5, calculate the volume generated when the shaded region is rotated 360° on the $x - axis$. [5 *marks*]

(a)

$$\int_5^6 6y - y^2 dy \text{ or } \frac{1}{2}(5)(5) \ \underline{\text{or}} \ \int_0^5 y\, dy$$

$$A_1 = \left[3y^2 - \frac{y^3}{3}\right]_5^6$$

Masukan had $\int_5^6$ ke dalam A_1

Total area = $\frac{8}{3} + \frac{25}{2}$ $(A_1 + A_2)$

$15\frac{1}{6}$ or $\frac{91}{6}$ units²

(b)

$$V = \pi\int_0^{12} x + 4 dx - \frac{1}{9}\pi\int_0^{12} x^2 dx$$

$$= \pi\left[\frac{x^2}{2} + 4x\right]_0^{12} - \frac{1}{9}\pi\left[\frac{x^3}{3}\right]_0^{12}$$

Masukan had $\int_0^{12}$ ke dalam A_1 atau A_2

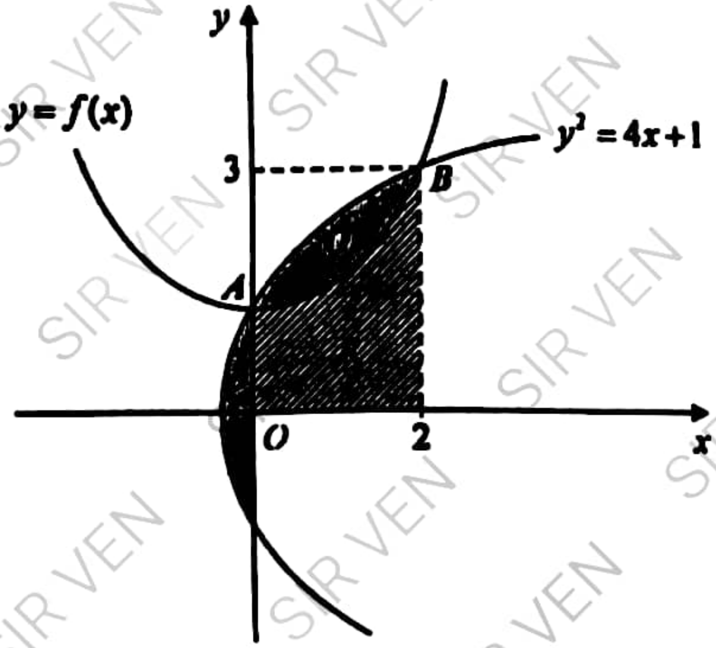
$= 120\pi - 64\pi$ $(A_1 > A_2)$

$= 56\pi$

Q15: SBP (ASRAMA)

Rajah 3 menunjukkan dua lengkung $y = f(x)$ dan $y^2 = 4x + 1$. Lengkung-lengkung tersebut bersilang pada titik A dan titik B.

Diagram 3 shows two curves $y = f(x)$ and $y^2 = 4x + 1$. The curves intersect at point A and point B.



Rajah 3

Diagram 3

(a) **Nyatakan koordinat A.**

State the coordinates of A.

[1 *markah*]

[1 *mark*]

(b) **Diberi bahawa $\int_0^2 f(x)dx = m$. Cari luas kawasan berlorek P dan Q dalam sebutan m.**

It is given that $\int_0^2 f(x)dx = m$. Find the area of the shaded region P and Q in terms of m.

[5 *markah*]

[5 *marks*]

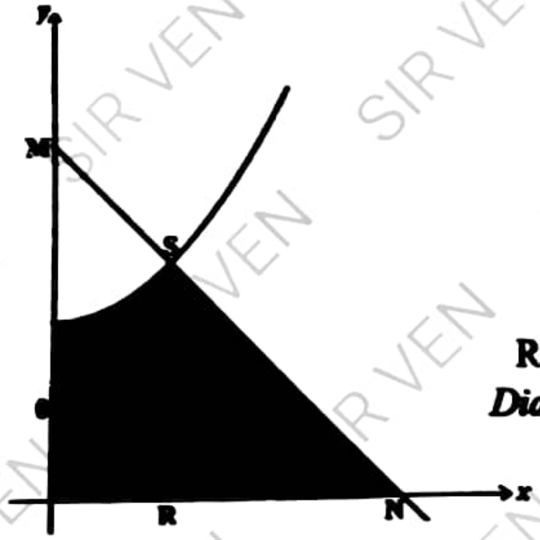
(c) **Kira isipadu janaan, dalam sebutan π, apabila rantau R dikisarkan $360°$ pada paksi-x jika luas keratan rentasnya ialah $\frac{1}{4}\pi(x^4 + 4x^2 + 4)$.**

Find the volume generated, in terms of π, when the region R is revolved $360°$ about the x-axis if the area of its cross-section is $\frac{1}{4}\pi(x^4 + 4x^2 + 4)$.

[4 *markah*]

[4 *marks*]

Q16: YIK KELANTAN



Rajah 6
Diagram 6

Dalam rajah 6, garis lurus MN ialah normal kepada lengkung $y=\frac{x^2}{3}+4$ pada titik $(3,5)$. Koordinat titik N ialah $(k,0)$. Garis lurus RS adalah selari dengan paksi-y

In Diagram 6, the straight line MN is normal to the curve $y=\frac{x^2}{3}+4$ *at point* $(3,5)$. *The coordinates of point N are* $(k,0)$. *The straight line RS is parallel to the* y*-axis*

Cari
Find

(a) nilai k, [3 *markah*]
the value of k, [3 *marks*]

(b) luas, dalam unit^2, kawasan berlorek, [4 *markah*]
the area, in unit^2, *of the shaded region,* [4 *marks*]

(c) isi padu yang dijanakan dalam sebutan π, apabila rantau yang dibatasi oleh lengkung, garis lurus RS, paksi-x dan paksi-y diputarkan 360° pada paksi-x.
volume generated in terms of π, *when the region bounded by the curve, straight line RS, the* x*-axis and the* y*-axis is revolved* 360° *about the* x*-axis.*

[3 *markah*]
[3 *marks*]

Q17: MRSM

(*a*) Diberi $\frac{d}{dx}\left(3x^2 - x\right) = f(x)$, cari nilai $\int_0^1 f(x)\, dx$. [3 *markah*]

Given $\frac{d}{dx}\left(3x^2 - x\right) = f(x)$, *find the value of* $\int_0^1 f(x)\, dx$. [3 *marks*]

(*b*) Fungsi kecerunan suatu lengkung adalah $3x - h$. Tangen kepada lengkung itu pada titik (5, 7.5) memotong paksi-x pada $x = 3.75$.
Cari persamaan lengkung itu. [3 *markah*]

The gradient function of a curve is $3x - h$. *Tangent to the curve at point* (5, 7.5) *cuts the x-axis at* $x = 3.75$.
Find the equation of the curve. [3 *marks*]

Jawapan / *Answer*: